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ON PLASMA LAYER INSTABILITY WITH THE NEUTRAL POINT OF A MAGNETIC FIELD

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ABSTRACT

It is shown in agreement with earlier-conducted studies [3, 4, 5] that the plasma layer equilibrium with a magnetic field containing a type-X neutral point is ustable. It is found that the Syrovatskiy's conclusion [9] ** on stability is erroneous on account of incorrect formulation of the problem and unacceptable assumptions.

* * *

COVER-TO-COVER TRANSLATION

Two viewpoints on flare generation are possible: The first consists in considering flares as occurring spontaneously, as a result of free field instability, frozen-in the solar plasma. The second views them as appearing as a result of action of outer fiedls (sunspot fields), compressing the plasma. The first possibility was considered by us — rather qualitatively — in [1], the second — more quantitatively — in [2]. It is not yet possible to state which of these two viewpoints is nearer the reality.

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* O neustoychivosti sloya plazmy s neytral'noy tochkoy magnitnogo polya.

** [NASA TT F-8376].

Following [1], we shall consider the symmetrical configuration of plasma equilibrium with the frozen-in field containing a type X neutral point (see Fig.1). The behavior of th lines of force is not dependent on z. The equation div $\mathbf{H} = 0$ is satisfied by the vector-potenial (0, 0, A) provided the field $\mathbf{H}$ is represented as

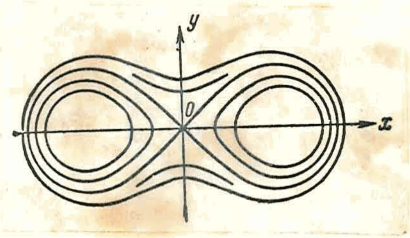

Fig. 1

$$\mathbf{H} = a \operatorname{rot} (\varphi, \nabla\psi), \tag{1}$$

where $\psi = z$, $A = \varphi$, a is a constant (see [3]). Further, the density of the current

$$\mathbf{j} = \frac{c}{4\pi} \operatorname{rot} \mathbf{H} = \frac{c}{4\pi} (0, 0, -\nabla^2 A), \tag{2}$$

and the equilibrium condition (rot $[\mathbf{j}, \mathbf{H}] = 0$) by the strength of symmetry configuration $(\mathbf{j}, \nabla)\, \mathbf{H} = 0$ will be

$$(\mathbf{H}\nabla j_z) = 0, \tag{3}$$

and inasmuch as $A =$ const along the line of force, we shall have according to (2)

$$\nabla^2 A = F(A), \tag{4}$$

where F is an arbitrary function of A. The solution of this equation

$$A = {}^1/_2 ax^2 + bxy + {}^1/_2 cy^2$$

g $H_x = bx + cy$, $H_y = -(ax + by)$. Thus, for example, the field distribution along the axis x will be

$$H \propto x, \tag{5}$$

which is admitted in [1] for the representation of the field near the neutral point. Let us note that for the considered configuration $H \to \infty$ for $x \to \infty$ and the equilibriun would require here an infinite density of current at the neutral point.

The considered configuration of field equilibrium is unstable (see the demonstration of it in ref [3, 4]): its lines of force are not interlocked (in any plane passing through the axis z, the number of "incoming" and "outgoing" lines of force is the same). The configuration does not satisfy the principle of line of force minimum length, along which $H^2/8\pi = \text{const}$ and approaches the state when the lines of force take the shape represented in Fig. 2. Plasma behavior near the neutral point o such type of configuration is qualitatively considered in [1], neglecting the curvature of the lines of force, and with the simplifying admission that the similitude is preserved at the beginning of compression, i. e. the distribution (5) is preserved. The same problem is considered more meticulously in [3] (para 6.4, with the same qualitative results. Finally in [5] it is shown that plasma hydrostatic equilibrium is unstable in the presence of a neutral point if gas pressure is lower than a certain value (cf. with the instability index

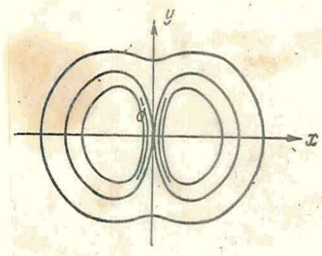

Fig. 2

$$\beta = \frac{n\kappa T}{H^2/8\pi} < 1,$$

obtained in [1).

Let us now examine the question of equilibrium stability of a plane-parallel plasma layer situated between the planes $x = -x_0$ and $x = +x_0$, with a type (5) field distribution or $H\left(0,\ H_0\frac{x}{x_0},\ 0\right)$ (the plane $x = 0$ is neutral), considering that inside the layer density and pressure are symmetrical functions of x, and outside the layer (vacuum), they are neglected. Such model of the state of plasma near the neutral po int has been accepted by us in [1] and [2]. Let us now examine the stability of such a state relative to small perturbations. We shall consider first of all for the qualitative approach the conductivity $\sigma = \infty$, and we shall neglect the shift current, considering the lines of force as straight and parallel at motion.

For such a case we have the following (Eiler) equations for small motions :

$$\frac{\partial \rho'}{\partial t} + \operatorname{div} \rho_0 \mathbf{v} = 0; \qquad (6.1)$$

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla \left(p' + \frac{H_0 H'}{4\pi}\right); \qquad (6.2)$$

$$\frac{\partial H'}{\partial t} = \operatorname{rot} [\mathbf{v} \times \mathbf{H}_0], \qquad (6.3)$$

where zero refers to equilibrium, and the stroke — to perturbations *. Considering the movement as adiabatic, and introducing the displacement (shifting) $\mathbf{s}$, so that $\mathbf{v} = \frac{\partial \mathbf{s}}{\partial t}$, we shall obtain

$$\rho' = -\operatorname{div} \rho_0 \mathbf{s}, \quad p' = -\frac{\gamma p_0}{\rho_0} \operatorname{div} \rho_0 \mathbf{s}, \quad \mathbf{H}' = \operatorname{rot} [\mathbf{s}, \mathbf{H}_0] \qquad (7)$$

and the perturbation of the full pressure will be

$$P' = p' + \frac{H_0 H'}{4\pi} = -\left(\gamma p_0 + \frac{H_0^2}{4\pi}\right) \frac{\partial \xi}{\partial x}, \qquad (8)$$

if we consider that shiftings take place only along the axis x and depend only on x ($s = \xi$). Introducing ξ and (8) into (6.2), and multiplying it by ξ and integrating from 0 to x_0, we shall obtain

$$\omega^2 \int_0^{x_0} \rho_0 \xi_1^2 \, dx = \left[\left(\gamma p_0 + \frac{H_0^2}{4\pi}\right) \xi_1 \frac{\partial \xi_1}{\partial x}\right]_0^{x_0} - \int_0^{x_0} \left(\gamma p_0 + \frac{H_0^2}{4\pi}\right) \left(\frac{\partial \xi_1}{\partial x}\right)^2 dx, \qquad (9)$$

having admitted the displacement $\xi = \xi_1 e^{\omega t}$.

For the investigation of instability by the sign ω^2, it is necessary to formulate the boundary conditions, particularly at the outer boundary of the plasma, where $\rho = 0$ and $p = 0$.

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* If the lines of force are being distorted, the term with H in (6.2) must be replaced by $\frac{1}{4\pi} [\operatorname{rot} \mathbf{H}', \mathbf{H}_0] + \frac{1}{c} [\mathbf{j}_0, \mathbf{H}']$,

where j_0 is the current density in equilibrium.

It is easy to see (cf [6]) that the requirement of current impulse continuity in our case amounts to conditions :

$$\{H_t\} = 0, \qquad \{p\} = 0, \tag{10}$$

where the brace indicates the jump, and t is the tangential component. The requirement of current energy continuity is reduced to the condition $\{v_t\} = 0$ by the strength of (10) and of the condition $\{H_n\} = 0$ (n being the normal), i.e. of velocity continuity. At the same time the condition $\{E_t\} = 0$ is identically satisfied. Passing from the displaced boundary, when (10) are fulfilled, to non-displaced $x = x_0$, we shall have with the precision to the small quantities of the first order

$$p_i' + \xi \frac{\partial p_{0i}}{\partial x} = 0; \quad H_i' + \xi \frac{\partial H_{0i}}{\partial x} = H_e' + \xi \frac{\partial H_{0e}}{\partial \xi} \quad \text{at} \quad x = x_0, \tag{11}$$

where indices i and e refer to the plasma and its surroundings (for $x = x_0$, $p_{oi} = 0$, $H_{oi} = H_{oe}$). According to (7), the first condition gives the identity $0 = 0$ ($\rho_0 = 0$), and the second —

$$H_{0i} \frac{\partial \xi}{\partial x} = -\left(H_e' + \xi \frac{\partial H_{0e}}{\partial x}\right). \tag{12}$$

If the conditions of the problem admit the representation $H_e' = H_{1e} e^{\omega t}$, then, considering $\xi = 0$ at $x = 0$, we obtain for the expression (9):

$$\omega^2 \int_0^{x_0} \rho_0 \xi_1^2 \, dx = -\frac{H_{0i}}{4\pi}\left(H_{1e} + \frac{\partial H_{0e}}{\partial x}\xi_{10}\right)\xi_{10} - \int_0^{x_0}\left(\gamma p_0 + \frac{H_0^2}{4\pi}\right)\left(\frac{\partial \xi_1}{\partial x}\right)^2 dx, \tag{13}$$

where ξ_{10} is the amplitude at the boundary. If $H'_e = 0$ (the surrounding field does not vary with time), then for the stability it is sufficient that the exterior field H_{0e} increase with the departure from the layer. As to the instability it may appear if H_{oe} decreases as it moves out of the plasma, i.e. if $\frac{\partial H_{0e}}{\partial x} < 0$. These signs are similar to those found in (7). If the surrounding field is uniform $\left(\frac{\partial H_{0e}}{\partial x} = 0\right)$, but the quantity $H'_e \neq 0$, then $\omega^2 = \frac{1}{H_e'} \frac{\partial H_e'}{\partial t}$ is determined by the growth or decrease of the outer field (or the sign of H'_e): for practically interesting cases (spot field variations) $\frac{\partial^2 H_e}{\partial t^2} > 0$ in the course of field growth ($H_{1e} > 0$) and compression ($\xi_1 < 0$) progresses with time ($\omega^2 > 0$), provided the field growth is sufficiently strong (the right-hand part of (13) is positive. This case of field's H_e increase, having exterior sources — the spots, is examined in [2].

Let us now deny ourselves the admissions $\sigma = \infty$, $\frac{\partial E}{\partial t} = 0$ and $(\mathbf{H}\nabla)\mathbf{H} = 0$ (the lines of force are parrelel lines) and let us consider the equilibrium stability of plasma layer with the field distribution, pressure and density (inside)

$$H_y = H_0 \frac{x}{x_0}, \quad p = p_0 \left(1 - \frac{x^2}{x_0^2}\right), \rho = \rho_0 \left(1 - \frac{x^2}{x_0^2}\right), \tag{14}$$

estimating $H_y = \text{const} = H_0$ beyond the plasma (this case practically coincides with the one considered in [2]. Let us consider that there is in equilibrium a current $\mathbf{J}_0 (0, 0, j_0)$ and a field $\mathbf{E}_0$ $(0, 0, E_0)$. Let us estimate that all the perturbations have a common multiplier

$$e^{ikz\omega t}, \tag{15}$$

while their amplitudes depend on x. In observing the equilibrium the constants H_0, p_0 and ρ_0 will be, according to (14) linked with the conditions

$$E_0 = \frac{j_0}{\sigma}, \; H_0 = \frac{1}{c} j_0 x_0, \; j_0 H_0 x_0 = 2p_0. \tag{16}$$

The problem of small oscillations in this case is considered in [8] (with the condition $\mathrm{Re}\,[\omega] > 0$) at correct boundary conditions $E_{xi} = E_{xe}$, $E_{zi} = E_{ze}$ and

$$p_{1i} - \frac{2p_0}{x_0\omega} v_1 = 0, \quad H_{1ye} + \frac{H_0}{x_0\omega} v_1 = H_{1yi} \tag{17}$$

(the last two are (11) and (12). It was shown in (8) that for the determination of stability at limit case $\sigma = \infty$, fundamental numbers of the equation

$$x_0^2 \frac{d^2\varphi}{dx^2} + 2x\psi \frac{d\varphi}{dx} = x_0^2 (k^2 + w^2 R^{-1} T) \varphi \tag{18}$$

must be found with boundary conditions

$$x = 0, \quad \varphi = 0; \; x = x_0, \; \varphi + \frac{2\alpha}{1 + 2\alpha} \frac{x_0}{\eta} \varphi' = 0, \tag{19}$$

where it is marked :

$$\psi = R^{-1} - 2aT^{-1}, \quad R = 1 + \frac{2}{\gamma}\frac{x^2}{x_0^2}, \quad T = 1 + 2\alpha\frac{x^2}{x_0^2}, \quad \alpha = \frac{1}{c^2}\frac{p_0}{\rho_0},$$
$$w^2 = \frac{\rho_0}{\gamma p_0}\omega^2, \quad \eta = x_0\left(k^2 + \frac{\omega^2}{c^2}\right)^{1/2}. \qquad (20)$$

The amplitude of all perturbations are then expressed through the function φ. In order to better visualize the behavior of the numbers in limit cases, we transform this problem to new variables. Introducing

$$z = \frac{\left(1 + \frac{2}{\gamma}\xi\right)^{\gamma/4}}{(1 + 2\alpha\xi)^{1/2}}\varphi; \qquad \xi = \left(\frac{x}{x_0}\right)^2, \qquad (21)$$

we obtain in place of (18), (19), the problem :

$$\frac{d}{d\xi}\left(s\frac{dz}{d\xi}\right) - pz - w^2x_0^2qz = 0; \qquad (22)$$
$$z = 0 \quad \text{at} \quad \xi = 0; \quad z' + \left[\frac{x_0}{2}\lambda' + a\right]z = 0 \quad \text{at} \quad \xi = 1, \qquad (23)$$

where the following designations are introduced :

$$s = 2\sqrt{\xi}; \quad q = \frac{1}{2\sqrt{\xi}}\,\frac{1 + 2\alpha\xi}{1 + \frac{2}{\gamma}\xi}; \qquad p = \frac{1}{2\sqrt{\xi}}(x_0^2k^2 + \Phi(\xi));$$
$$\Phi(\xi) = \psi + \xi\psi^2 + \sqrt{\xi}\psi'; \quad a = \frac{4\alpha - \gamma}{2(\gamma + 2)(1 + 2\alpha)};$$
$$\lambda' = \frac{1 + 2\alpha}{2\alpha}\left(k^2 + \frac{\omega^2}{c^2}\right)^{1/2}. \qquad (24)$$

Multiplying (22) by z and integrating along ξ from 0 to 1, we shall obtain by the strength of boundary conditions (23) :

$$\omega^2x_0^2 + \int_0^1 pz^2d\xi = -(2a + x_0\lambda')z_1^2 - \int_0^1 sz'^2d\xi, \qquad (25)$$

if we adopt the normalization $\int_0^1 qz^2d\xi = 1$. Let us designate

$$\Lambda^2 = k^2 + \frac{\omega^2}{c^2}, \quad \int_0^1 \frac{z^2d\xi}{2\sqrt{\xi}} = B, \quad \int_0^1 \frac{\Phi z^2}{2\sqrt{\xi}}d\xi + \int_0^1 sz'^2d\xi + 2az_1^2 = A; \qquad (26)$$

then

$$w^2 = \frac{1}{\alpha\gamma}(\Lambda^2 - k^2)$$

and instead of the equation for ω^2 we shall obtain the following equation for Λ^2:

$$\Lambda^2 + \frac{\gamma z_1^2 (1+2\alpha)}{2x_0} \Lambda + k^2 (\alpha\gamma B - 1) + \frac{\alpha\gamma}{x_0^2} A = 0. \tag{27}$$

The solution of this equation is

$$\Lambda = -\frac{\gamma z_1^2 (1+2\alpha)}{4x_0} \pm \left[\frac{\gamma^2 z_1^4 (1+2\alpha)^2}{16 x_0^2} + k^2 - \alpha\gamma \left(k^2 B + \frac{A}{x_0^2} \right) \right]^{1/2}. \tag{28}$$

The quantity is of the order of the square of the speed of sound in the plasma ratio to the speed of light, quantity very small ($\simeq 10^{-8}$), and that is why we have with great precision

$$\Lambda = -\frac{\gamma z_1^2}{4x_0} \pm \left[\frac{\gamma^2 z_1^4}{16 x_0^2} + k^2 \right]^{1/2} \tag{29}$$

(the quantities A, B, and z_1 are of the order of the unity for the adopted normalization). Therefore, Λ is always real and for $\Lambda^2 < k^2$ we shall have stability, and for $\Lambda^2 > k^2$ – instability. For k = 0, case of strictly longitudinal compressions – plane waves, see above, we have $\Lambda_1 = 0$ $\frac{\omega_2^2}{c^2} > 0$, $\Lambda_2 = -\frac{\gamma z_1^2}{2x_0}$, $\Lambda_2^2 =$ i.e. the instability (at corresponding boundary conditions). When $k \neq 0$ (narrowing along the axis z) we have from the expressions for ^{2} (26) and (29):

$$\omega_1^2 = 2\left(\frac{\gamma z_1^2 c}{4x_0}\right)^2 \left\{1 - \left[1 + k^2 \left(\frac{4x_0}{\gamma z_1^2}\right)^2\right]^{1/2}\right\},$$
$$\omega_2^2 = 2\left(\frac{\gamma z_1^2 c}{4x_0}\right)^2 \left\{1 + \left[1 + k^2 \left(\frac{4x_0}{\gamma z_1^2}\right)^2\right]^{1/2}\right\}, \tag{30}$$

i.e. there is also instability for any value of k. Therefore, a plane-parallel plasma layer, containing a neutral point of the considered type (with distribution of the type (4)) is unstable. Let us note that for the estimate in [8] of the effect of finite conductivity on oscillations, the solution for ω_1 was adopted.

For the solar plasma, when the characteristic dimensions of x_o is comparable with **c**, the quantities ω_1 will be $\sim z_1^2$ (~ 1 at selected normalization). The charactristic times of a rapid flare development are of the order of several seconds.

Therefore, if one considers a flare as a result of instability of the considered layer, it is obvious that the stable (or rather stationary, to be more precise) state prior to the flare ought to be already characterized by another distribution of density and field (for example, by field growth at $|x| > x_o$), and the flare itself should by the same token be viewed as a result of field re-distribution on account of hydromagnetic movements (in case of a free field) or of variation of spot position (or spot field intensity) in case of an outer field.

It may be seen from (29) that had we utilized an incorrect boundary condition $z_1 = 0$, we would have obtained from (28) $\Lambda^2 \simeq k^2 - \alpha\gamma \frac{A}{x_0^2}$, i. e. $\omega^2 < 0$, and would have drawn a conclusion of stability, as this was done in [9]. The same prudence must be exercized at transition of $x_0 \to \infty$, for the solution $z(\xi)$ includes x_o as parameter (see (24)) and without investigating the convergence of integrals A and B at $x_o \to \infty$, nothing can be said about the numbers ω^2. Generally speaking, as is well known, the character of eigen-values for a problem with a finite boundary may radically differ from the same for an infinite boundary.
It may be seen from (9), that at field distribution (5) the point $x = \infty$ is a peculiar point of the equation and the integrals in (9) have sense if ξ_1 does not decrease more slowly than $1/\sqrt{x}$ at $x \to \infty$. This is linked with the fact that at $x = \infty$ we have to do with the configuration of an infinitely great potential energy $\left(\int \frac{H^2}{8\pi} dx\right.$ diverges at field distribution $\left.(5)\right)$, where the problem of minimum energy itself (i. e. of stability) loses its sense without special reservations.

That is why it is senseless, for example, to speak of equilibrium stability of a boundless nonuniform medium in the general, without stating what field and plasma density distributions are realized in equilibrium near the neutral point, if, obviously, we do not have in mind the well-known problem of magneto-acoustic waves considered in [9]. By the way, let us note that inasmuch as for solar flares $(v/v_T)^2 \gtrsim\sim 10^2$ (v being the characteristic plasma velocity, v_T — the thermal velocity), the neglecting of the inertial term in the equation of motion made in [9] is unacceptable. Neither is justified the assertion in [9] of the impossibility of self-modeled compression, for the total volume elasticity is

$$\gamma p + \frac{H^2}{4\pi} = 2p_0\left(1 - \frac{1}{6}\frac{p}{p_0}\right)$$

quantity practically invariable inside the layer in equilibrium (p_0 is the pressure at the neutral point, $p/p_0 \leqslant 1$, $\gamma = 5/3$), so that the compression is nearly everywhere uniform (if it is strictly avross the field).

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R E F E R E N C E S

1. A. B. SEVERNYY. Izv. Krymsk.Astrofiz.Obs., 20, 22, 1958.

2. A. B. SEVERNYY. Astronom.Zh. 38, 402, 1961., Izv.Kr.Astrofiz. Observatorii, 27, 71, 1962.

3. D. DANZHI. Kosmicheskaya elektrodinamika, para 4, 3, M., 1961.

4. D. DANZHI., Ibid. para 3, 7; see also Phil. Mag. 44, 725, 1953.

5. P. SWEET. Electrom. Phenom.in Cosmic Physics. Cambr, p.123, 1958.

6. L. D. LANDAU and E. M. LIFSHITS. Elektrodin.spl.sred GTTI, M.1957

7. S. I. BRAGINSKIY, B. B. KADOMTSEV, Fizika plasmy i.t.d.t. III, 300, Izd-vo A. N.SSSR, 1958.

8. M. KRUSKAL, and M. Schwartzschild. Proc. Roy.Soc. (A), 223, 348, 1954.

9. S. I. SYROVATSKIY., current issue A.Zh., 39, 6, p.987, 1962. (NASA TT F-8376)